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EXPERIMENTAL IMPLEMENTATION OF MIGRATIONS IN MULTIPLE-ATTRACTOR SYSTEMS

R. WEIGEL and E. ATLEE JACKSON
Department of Physics, 1110 West Green Street and
Center for Complex Systems Research, Beckman Institute,
University of Illinois at Urbana-Champaign,
Urbana, IL 61801, USA

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It was shown by Jackson and Grosu [1995a] that entrainment control of a complex dynamic system could be achieved by using an open-plus-closed-loop (OPCL) control action. Jackson [1995] showed that, for multiple-attractor systems, it is also possible to transfer the dynamics of a system between arbitrary attractors, using this control only for a limited period of time ("migration control"). In the case of the Chua system [1986], Jackson and Grosu [1995b] showed that this migration action can be achieved using only a limited pre-recorded signal of one variable of the "target" attractor. Here we study the experimental implementation of this migration action on a Chua circuit with five attractors (fixed points, a limit cycle, and chaotic attractors) [Wu & Pivka, 1994]. It is shown that reliable migrations are possible with a pre-recorded signal of one target variable, provided that the signal is longer than a minimal time, dependent on an adjustable control strength. The minimal time for successful migration is studied for all distinct migration processes. It is also shown that this action is robust to the limited accuracy of the recorded data.

1. Introduction

It seems quite likely that, in order for a complex system to be capable of discriminating various characteristics of incoming (sensory) information, it must possess a repertoire of stable (or possibly, meta stable) dynamic states. That is, such "functional" dynamic systems must be multiple-attractor systems (MAS). Only in this case can the system respond in a dynamically-resonant fashion with selected sensory inputs. It is therefore of considerable interest to obtain some insights into methods that can reliably transfer a dynamic state of an MAS from one attractor to another. There are, of course, a variety of questions that can be raised in this connection, depending on whether the methods involve endogenous or exogenous actions, however it may be hoped that understandings in one area will yield insights in the other.

The present study considers external actions that can reliably transfer an MAS from any of its attractors A_i to any other attractor, A_j. This transfer is referred to as a "migration" action on the system [Jackson, 1990]. While this concept is not new, it was only with the discovery of the open-plus-closed-loop (OPCL) interaction method [Jackson & Grosu, 1995a] that a general reliable interaction capability has been devised for this migration action [Jackson, 1995].

In the search for a simple MAS to demonstrate these ideas, it was found that the Chua system can possess at least five distinct dynamic attractors [Wu & Pivka, 1994; Jackson, 1995]. Most importantly, it was realized that a migration action, $A_i \to A_j$, in this system can be achieved by only having a short recording of the dynamics of one variable of the target attractor, A_j [Jackson & Grosu, 1995b]. This is a special case of a more general class of systems, in which such transferals between attractors can be accomplished without any dynamic model of the MAS. While the theory was quite clear in this matter, there were several practical matters that could only be established by an experimental investigation of this process. This report shows the experimental feasibility of this method and its robust character in the light of experimental limitations on data accuracy and noise.

2. Migration Actions Applied to the Chua System

The equations of motion of the Chua system [Chua *et al.*, 1986] written in dimensionless form are

$$\frac{dx}{d\tau} = \alpha(y - x - f(x)) \quad (1)$$

$$\frac{dy}{d\tau} = x - y + z \quad (2)$$

$$\frac{dz}{d\tau} = -\beta y - \gamma z \quad (3)$$

where $f(x)$ is a piecewise linear function given by

$$f(x) = bx + \frac{1}{2}(a - b)(|x + 1| - |x - 1|)\,. \quad (4)$$

The present migration action is based on the use of a pre-recorded signal from each of the five attractors, which are used as the "goal" dynamics, $g(t)$, in the OPCL method. Because of this selection, the Hübler component of the OPCL action vanishes [Jackson & Grosu, 1995a], leaving only the linear feedback component, $C(g, t)(g(t) - x(t))$. For autonomous equations, $dx/dt = F(x)$, the matrix $C(g)$ is given by $C(g) = dF(g)/dg - A$, where A is an arbitrary constant matrix, whose eigenvalues all have negative real parts. Because the only nonlinearity of the Chua system is $f(x)$ in Eq. (1), the matrix A can be selected so as not to involve a feedback action in y or z [Jackson & Grosu, 1995b]. In dimensionless form, the controlled system involves only replacing (1) with

$$\frac{dx}{d\tau} = \alpha(y - x - f(x)) + \left[\alpha\frac{R}{R_c}(g - x)\right] S(t) \quad (5)$$

$$\frac{R}{R_c} = -\left[\left(1 + \frac{df}{dx}\right) + pR\right] \quad (6)$$

where p is the adjustable OPCL control parameter from the matrix A, R_c is the experimental control parameter, and (2) and (3) are unchanged. In terms of experimental variables, the controlled equations read

$$C_1\frac{dv_1}{dt} = \frac{1}{R}(v_2 - v_1) - f(v_1) + \frac{1}{R_c}(g_1 - v_1) \quad (7)$$

$$C_2\frac{dv_2}{dt} = \frac{1}{R}(v_1 - v_2) + i_L \quad (8)$$

$$L\frac{di_L}{dt} = -(v_2 + i_L r_o) \quad (9)$$

$$f(v_1) = m_o v_1 + \frac{1}{2}(m_1 - m_o)(|(v_1 + BP)| - |(v_1 - BP)|) \quad (10)$$

3. Experimental Implementation

A schematic diagram or the experimental circuit used to implement the migration control is shown in Fig. 1. The circuit components used are (C_1, C_2, L, r_o, R) = (0.482 μF, 5.20 μF, 114.2 H, 11.0 Ω, 512 Ω), and the Chua diode characteristics are given by ($\pm$BP, m_o, m_1)=($\pm$0.46 volts, $-1.3 \times 10^{-4}\ \Omega^{-1}$, $-2.1 \times 10^{-3}\ \Omega^{-1}$). The recording of g_1 was made to a 12 bit analog input board (Nat. Inst. NB-MIO-16) and stored in a computer. This recording was then played back as a 12 bit analog signal as shown in Fig. 1. When the playback of the goal is initiated, the switch is opened and remains open for a predetermined length of time. A schematic of the switch is shown in Fig. 2. When v_s is high ($S(t) = 1$), the resistance is 60 Ω and when v_s is low ($S(t) = 0$), the resistance is 10^{12} Ω and the control is isolated from the Chua circuit. The timing is achieved by using a 555 timer which is triggered by the beginning of the playback of g_1.

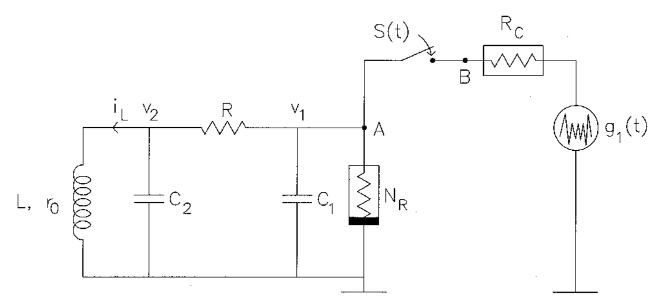

Fig. 1. Schematic diagram of Chua circuit and experimental implementation of migration action. $S(t)$ is the control switch and R_c is the control resistance through which the migration control signal is played back to the system.

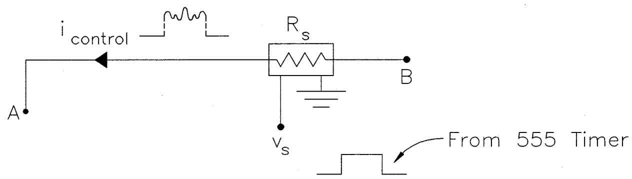

Fig. 2. Schematic of the switch, $S(t)$. The switch uses a CMOS Quad Bilateral Switch (CD4066A) with open switch resistance $R_s = 60\ \Omega$ when the switch trigger voltage v_s is high and $R_s = 10^{12}\ \Omega$ for v_s low.

As can be seen from the migration figures, the 12 bit resolution of the recording created small fluctuations in the signal. Upon recording, the resolution is 0.005 volts and upon playback, the resolution is 0.005 volts which gives a total uncertainty in the control signal of 0.01 volts. The experiments in this paper demonstrate that the migration control is robust to fluctuations and uncertainties in the playback of the signal.

4. Experimental Results

The five attractors of the Chua system are shown projected onto the (x, y) plane in Fig. 3. To apply migration control, the system was first put into an attractor, and a recording of v_1 was made. Next, the system was put into one of the remaining four attractors. The goal was then played back to the system for a certain length of time corresponding to the time $S(t) = 1$ in (4) (otherwise $S(t) = 0$). This process was repeated 10 times which corresponds to 10 different initial conditions of the dynamics when the control is initiated. It was found that above a threshold time, $t_{\min}$, all 10 playbacks resulted in a successful migration (system remained on the goal attractor after the switch was opened, $S(t) = 0$). This time $t_{\min}$ was studied as a function of the control resistance and any migration sequence $A_i \to A_j$. The control times are all normalized to the "period" of the system while it is in the chaotic attractor where the period is defined as the average time between peaks in the v_1 time series and has the value 0.0066 sec and represents a characteristic oscillation time. A typical example of this is shown in Fig. 4. Notice that below $t_{\min}$ successful migrations can occur when the control is shut off if the system happens to be in the basin of attraction of the goal attractor. Above $t_{\min}$ the control takes the dynamics close enough to the attractor such that when it is shut off the system remains on the attractor.

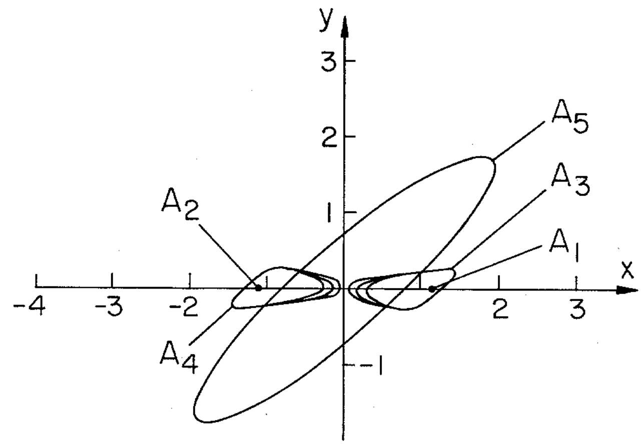

Fig. 3. Schematic of the five attractors of the Chua system for the dimensionless parameters used in this experiment projected onto the normalized (x, y) plane. [See Eqs. (1)–(4)] Attractor 5 has been scaled down for clarity.

Figure 5 shows this minimum control time versus the control resistance for all transferals excluding those to the limit cycle. The transferals to the limit cycle are not considered because outside of a small sphere which encloses the other four attractors ($A_1 - A_4$), all initial conditions approach the limit cycle (A_5). Therefore, the minimum control time will not reflect the fact that the control action is bringing the dynamics close to the goal, but rather only shows that the migration action moves the dynamics into a large basin of attraction. This is in contrast to the migrations considered in Fig. 5 in which the migration action must put the dynamics very close to the goal dynamics before the switch is opened because the basins of attraction of $A_1 - A_4$ are intertwined. Above the last control resistance value plotted, the migration control is not reliable for any control time above 50 cycles for any initial condition.

Several phase space representations of the migration are shown in Figs. 6 and 7. In Fig. 7, the control time is greater than $t_{\min}$, and when the switch opens, the system is close enough to the goal attractor that it remains on that attractor. A migration from chaotic attractor A_3 to chaotic attractor A_4 is shown in Fig. 7.

Finally, in Fig. 8 we consider the time series of v_1 versus g_1. The system is on the attractor A_5 when the control sequence $g_1(t) = A_2 \to A_1 \to A_4 \to A_3 \to A_5$ is initiated. As can be seen from the graph, $|v_1 - g_1|$ approaches zero quite rapidly.

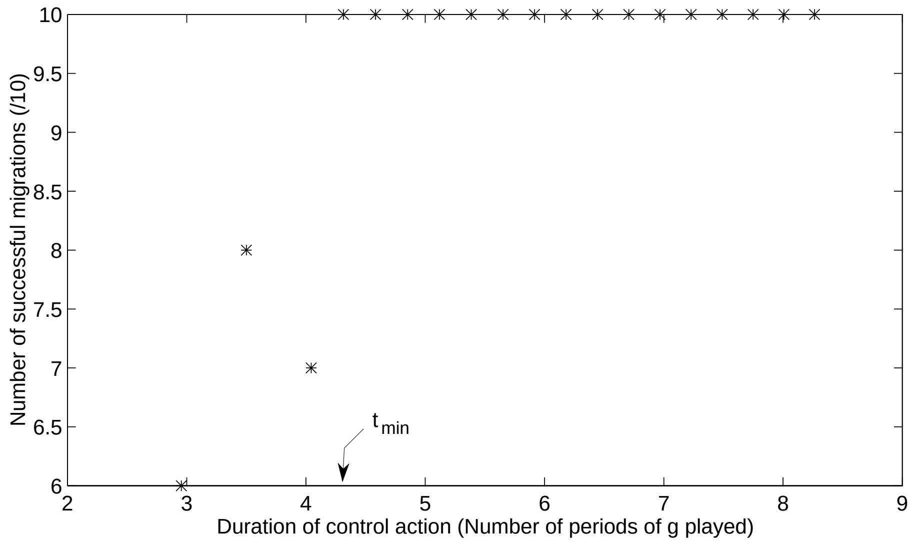

Fig. 4. Representative example of the behavior of the migration action for various control times indicating the minimum time for reliable migration. (Migration is from A_3 to A_4 with $R_c = 358.8\ \Omega$.)

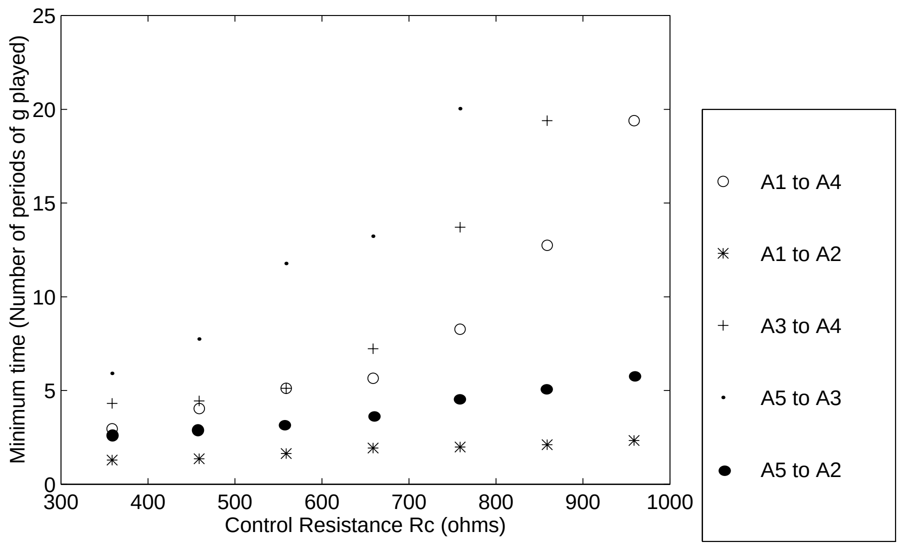

Fig. 5. Behavior of the minimum control time versus the control resistance for all distinct types of migrations.

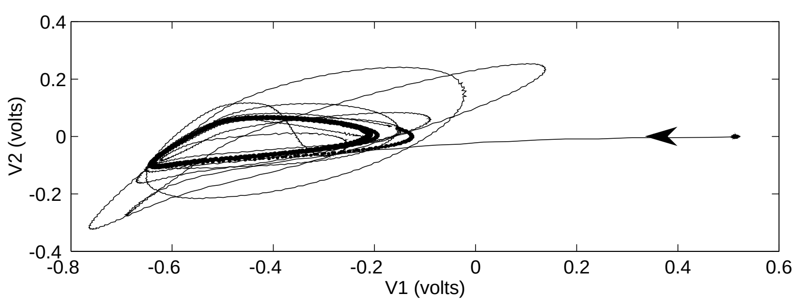

Fig. 6. Illustration of a successful migration from A_1 to A_4 (control duration greater than $t_{\min}$). The heavy line is the dynamics when $S(t) = 0$. The axes reflect actual measured voltages corresponding to Eqs. (7)–(10).

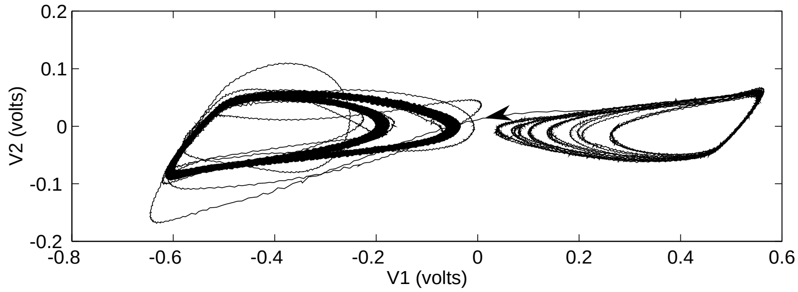

Fig. 7. The migration between the two chaotic attractors from A_3 to A_4. The heavy line is the dynamics when $S(t) = 0$.

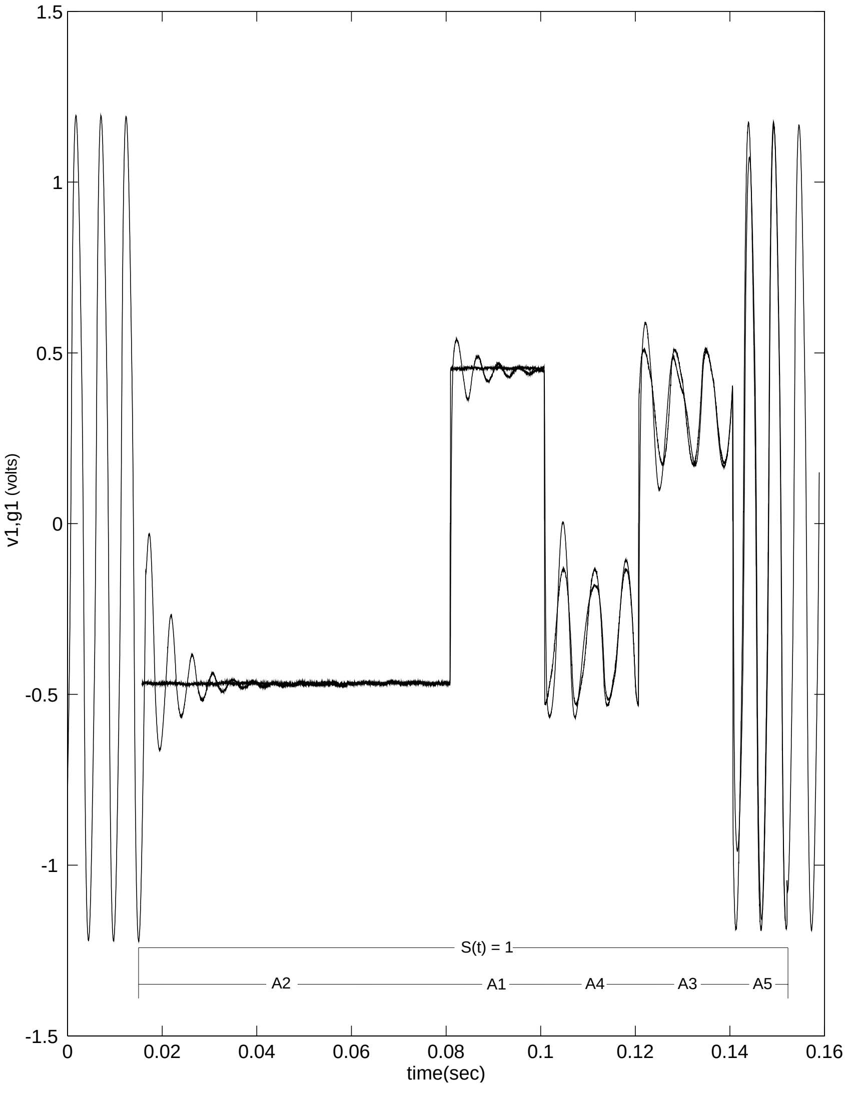

Fig. 8. Evolution of the dynamics using a changing sequence $g(t) = A_2 \to A_1 \to A_4 \to A_3 \to A_5$ with $S(t) = 1$ during this process. The control resistance is 360 Ω.

5. Conclusion

This study has shown that the theoretical migration action proposed by Jackson and Grosu [1995b], which only makes use of short pre-recorded signals of some variable dynamics in the target attractors, rather than using knowledge of the dynamic equations, can be experimentally implemented in a robust fashion. The migration can be accomplished with limited accuracy of the recorded data and control parameters, and with influences of noise and short migration control action times. Moreover, this study showed that the migration is successful even though details of the system dynamics are ignored as a result of the migration control action containing information from a recording of only one of the dynamic variables. The generalization of the exogenous application of this migration method to more complex systems involving removal from destructive chaotic modes of dynamics, such as fibrillations, and spatial-temporal forms of neurological seizures, is a challenge for future research. In addition, this may lead to concepts about how a system can randomly search through its inherent collection of multiple attractors, in order to endogenously respond in an adaptive fashion to changes in environmental inputs. This may someday give insights into the ideas of chaotic dynamic searches within the neural networks of the brain [Freeman, 1991, 1992]

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